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Brief communication

Size-dependent vibration characteristics of fluid-conveying microtubes

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Abstract

In this paper, a new theoretical model is developed, based on the modified couple stress theory, for the vibration analysis of fluid-conveying microtubes by introducing one internal material length scale parameter. Using Hamilton's principle, the equations of motion of fluid-conveying microtubes are derived. After discretization via the Differential Quadrature Method (DQM), the analytical model exhibits some essential vibration characteristics. For a microtube in which both ends are supported, it is found that the natural frequencies decrease with increasing internal flow velocities. It is also shown that the microtube will become unstable by divergence at a critical flow velocity. More significantly, when the outside diameter of the microtube is comparable to the material length scale parameter, the natural frequencies obtained using the modified couple stress theory are much larger than those obtained using the classical beam theory. It is not surprising, therefore, that the critical flow velocities predicted by the modified couple stress theory are generally higher than those predicted by the classical beam theory.

Keywords: Fluid-conveying microtube; Vibration; Modified couple stress theory; Size effect

1. Introduction

Microbeams/nanobeams have become one of the major structures used widely in micro-electronic-mechanical systems (MEMS) and nanotechnology, such as those employed in sensors, actuators, fluid storage, fluid transport and drug delivery [see, e.g., Moser and Gijs (2007), De Boer et al. (2004), Yoon et al. (2005), Reddy et al. (2007), He et al. (2008), Lee and Chang (2008), Kuang et al. (2009)]. In such applications, the thickness of the beam-type structures is typically on the order of microns or even nanometers (Park and Gao, 2006). In a recent paper by Rinaldi et al. (2010), the inside diameter of the circular microtubes considered ranged from 1 to 100 μm. It was reported that microtubes/microbeams containing an internal fluid flow exist in a class of microresonators [see, e.g., Najmzadeh et al. (2007), Enoksson et al. (1997), Sparks et al. (2009)]. It is not surprising, therefore, that the topic of fluid transport through beams (or tubes) is now of considerable interest for potential micro- and nano-fluidic device applications (Whitby and Quirke, 2007).

It seems straightforward to directly extend the analysis of macroscale structures to that of microscale structures. This is not so, however. In fact, in the last 10 years, size-dependent behavior of microscale structures has been observed

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experimentally. Experimental work on this topic appears to have started in the 1990s. Some of the key contributions in this area were made by Fleck et al. (1994), Ma and Clarke (1995), Stolken and Evans (1998), Chong and Lam (1999), Lam et al. (2003), and McFarland and Colton (2005). The size dependence phenomenon has been observed in materials made of both metals and polymers (such as copper, single silver crystals, nickel, epoxy polymers, etc.). As an example, in the bending testing of polypropylene micro-cantilevers, McFarland and Colton (2005) observed that the measured stiffness values are at least four times larger than those predicted by the classical beam theory, and the deformation is also in the linear and elastic region. These experimental results demonstrate that size dependence is intrinsic to certain materials with microstructures.

As reported by Govindjee and Sackman (1999) and Wang (2009), although the classical continuum theory is relevant to some extent, internal material length scales are often small enough to call the applicability of classical continuum theory into question. For quite a number of materials, therefore, the classical continuum theory (conventional strain-based mechanics theory) may be inadequate for predicting the response of microstructures, and the utilization of higher order continuum theories containing internal material length scale parameters is inevitable.

The classical couple stress elasticity theory (Mindlin, 1964; Toupin, 1962; Mindlin and Tiersten, 1962) is a higher order continuum theory that contains four material constants (two classical and two additional) for isotropic elastic materials. This theory was used to study the length scales in the static and dynamic torsion of a circular cylindrical micro-bar by Zhou and Li (2001) and was utilized to model pure bending of cylinders by Anthoine (2000). Another important higher order continuum theory is the so-called non-local elasticity theory. The non-local elasticity theory, which has been widely applied to analyze nanostructures, also contains non-classical material constants [see, e.g., Eringen (1983)].

Recently, a modified couple stress theory was developed by Yang et al. (2002), in which the couple stress tensor is symmetric and only one internal material length scale parameter is involved, unlike those in the above-mentioned classical couple stress theory. The modified couple stress theory has been used to study the mechanical and dynamical behavior of microbeams in the past [see, e.g., Kong et al. (2008), Ma et al. (2008), Park and Gao (2006)]. More importantly, the bending rigidity of epoxy polymeric beams predicted by the modified couple stress theory agrees well with that obtained experimentally (Park and Gao, 2006).

For fluid-conveying tubes/pipes/beams, to the author's knowledge, the analytical models developed thus far have used the conventional strain-based mechanics theories [see, e.g., Païdoussis (1998), Modarres-Sadeghi and Païdoussis (2009), Wadham-Gagnon et al. (2007), Païdoussis et al. (2007, 2008), Modarres-Sadeghi et al. (2007), Kuiper and Metrikine (2008), Kuiper et al. (2007)]. As mentioned in the foregoing, the conventional strain-based mechanics theories can sufficiently capture the essential vibration characteristics of fluid-conveying tubes with a large length scale. However, for quite a number of materials, as stated previously, the conventional strain-based mechanics theories are not capable of predicting the microstructure-dependent size effect when the tube size is on the micron scale. Therefore, accurate characterization of the dynamical behavior on the micron scale is vital for reliable and optimal design of microtubes for micro-fluidic device applications.

The objective of the present paper is to establish a theoretical model for fluid-conveying microtubes using the modified couple stress theory. The tube material is assumed to obey the modified couple stress theory as developed by Yang et al. (2002). The equation of motion, in which an internal material length scale parameter is included, will be derived by using Hamilton's principle. Based on the derived equation of motion, the vibration and instability of the microtube will be studied. It will be shown that the size effect on natural frequency and on critical flow velocity is significant. The difference between the tube model presented here and the classical tube model based on the classical Bernoulli–Euler beam theory will be quantitatively shown and analyzed.

2. Derivation of the equation of motion

The system under consideration consists of a uniform tubular microbeam of length L, external cross-sectional area A_p, mass per unit length m, conveying incompressible fluid of mass per unit length M, flowing axially with velocity V. The internal cross-sectional flow area is A_f. The cross-section of the microtube is symmetric, either circular or rectangular. The Cartesian axes for a planar tube analysis are established, as shown in Fig. 1. The x-axis is coincident with the centroidal axis of the microtube.

The use of the modified couple stress theory for microbeams will be reviewed first. For more details on this theory, the interested reader is referred to Yang et al. (2002) and Kong et al. (2008). According to the modified couple stress theory, the strain energy density is a function of both the strain (conjugated with stress) tensor and the curvature (conjugated with couple stress) tensor. Therefore, the strain energy U in a deformed isotropic linear elastic material

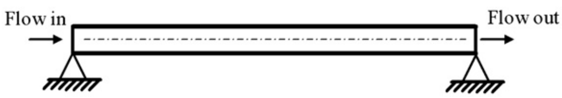

Fig. 1. Schematic of a fluid-conveying microtube in which both ends are positively supported.

occupying region Ω can be written as

$$U = \frac{1}{2}\int_{\Omega}(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})\mathrm{d}v \quad (i,j = 1,2,3). \tag{1}$$

In the above equation, the stress tensor σ_{ij}, the strain tensor ε_{ij}, the deviatoric part of the couple stress tensor m_{ij}, and the symmetric curvature tensor χ_{ij}, are given by

$$\sigma_{ij} = \lambda \mathrm{tr}(\varepsilon_{ij})\delta_{ij} + 2G\varepsilon_{ij}, \quad \varepsilon_{ij} = \frac{1}{2}[\nabla u_i + (\nabla u_i)^{\mathrm{T}}], \tag{2,3}$$

$$m_{ij} = 2l^2 G\chi_{ij}, \quad \chi_{ij} = \frac{1}{2}[\nabla\theta_i + (\nabla\theta_i)^{\mathrm{T}}], \tag{4,5}$$

respectively, where λ and G are Lamé's constants (G is also known as the shear modulus), δ_{ij} is Kronecker's delta function, l is a material length scale parameter, u_i is the displacement vector, and θ_i the rotation vector given by

$$\theta_i = \frac{1}{2}\mathrm{curl}(u_i). \tag{6}$$

It is noted that both σ_{ij} and ε_{ij} are symmetric. From Eq. (4), it can be seen that the square of the length scale parameter l is the ratio of the curvature modulus to the shear modulus. Therefore, l may be viewed as a material property representing the effect of couple stress.

Now, according to the Bernoulli–Euler beam theory, the displacement components can be written as

$$u = -z\psi(x,t), \quad v = 0, \quad w = w(x,t), \tag{7}$$

where u, v, and w are the displacement components in the x-, y-, and z-directions, respectively; $\psi(x)$ is the rotation angle of the centroidal axis of the microtube. For small deformations, $\psi(x)$ may be given by

$$\psi(x) \approx \frac{\partial w(x,t)}{\partial x}. \tag{8}$$

The combination of Eqs. (3), (7), and (8) yields

$$\varepsilon_{xx} = -z\frac{\partial^2 w(x,t)}{\partial x^2}, \quad \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yz} = \varepsilon_{zx} = 0. \tag{9}$$

Similarly, the combination of Eqs. (6)–(8) yields

$$\theta_y = -\frac{\partial w(x,t)}{\partial x}, \quad \theta_x = \theta_z = 0. \tag{10}$$

Substitution of the above equation into Eq. (5) gives

$$\chi_{xy} = -\frac{1}{2}\frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2}, \quad \chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{yz} = \chi_{zx} = 0. \tag{11}$$

Then, the substitution of Eq. (9) into Eq. (2) gives

$$\sigma_{xx} = -Ez\frac{\partial^2 w(x,t)}{\partial x^2}, \quad \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0, \tag{12}$$

where E is Young's modulus of the tube material, which is related to Lamé's constant λ and Poisson's ratio μ. In the above equation, the Poisson effect is neglected in order to facilitate the formulation of a simple beam theory.

Similarly, substitution of Eq. (12) into (4) gives

$$m_{xy} = -Gl^2 \frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2}, \quad m_{xx} = m_{yy} = m_{zz} = m_{yz} = m_{zx} = 0. \tag{13}$$

Substituting Eqs. (9) and (11)–(13) into Eq. (1), one obtains

$$U = \frac{1}{2}\int_0^L (EI + GA_p l^2)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \mathrm{d}x, \tag{14}$$

where I is the usual second moment of cross-sectional area of the tube.

The kinetic energy of the tube is

$$T_p = \frac{m}{2}\int_0^L \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}x. \tag{15}$$

Furthermore, the fluid kinetic energy is given by (Païdoussis, 1998)

$$T_f = \frac{M}{2}\int_0^L \left[\left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right)^2 + V^2\right] \mathrm{d}x \tag{16}$$

Now, according to the formulation of Benjamin (1961), the statement of Hamilton's principle for a fluid-conveying microtube can be written as

$$\delta \int_{t_1}^{t_2} (\mathcal{L} - MV^2 u_L)\mathrm{d}t - \int_{t_1}^{t_2} MV(\dot{w}_L + V w_L')\delta w_L \, \mathrm{d}t = 0, \tag{17}$$

where the Lagrangian $\mathcal{L} = T_p + T_f - U$ and the subscript L represents the values of the corresponding quantities at $x = L$; the overdot denotes differentiation with time, and $()' = \partial()/\partial x$.

For a fully supported microtube, not allowing any axial sliding at $x = L$, since $u_L = w_L = 0$, Eq. (17) reduces to

$$\delta \int_{t_1}^{t_2} \mathcal{L} \, \mathrm{d}t = 0. \tag{18}$$

Substituting expressions (14)–(16) into the above equation and applying the usual variational techniques to Eq. (18), the equation of lateral motion is obtained as

$$(EI + GAl^2)\frac{\partial^4 w}{\partial x^4} + MV^2\frac{\partial^2 w}{\partial x^2} + 2MV\frac{\partial^2 w}{\partial x \partial t} + (M + m)\frac{\partial^2 w}{\partial t^2} = 0. \tag{19}$$

The boundary conditions for a pinned–pinned microtube can be written as

$$\frac{\partial^2 w(0,t)}{\partial x^2} = w(0,t) = 0, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = w(L,t) = 0. \tag{20}$$

It can be seen from Eq. (19) that the equation of motion of the microtube is related to two parts: one associated with M, m, V, and EI as in the classical tube model and the other associated with GAl^2. Therefore, the current tube model based on the modified couple stress theory contains only one additional material constant in addition to the four classical parameters. Furthermore, when the size effect is suppressed by letting $l = 0$, the new model will reduce to the classical tube model.

3. Method of solution

Various methods for solving Eq. (19) combined with the boundary condition (20) may be found in, for example, Païdoussis (1998), Kuiper and Metrikine (2005), and Wang et al. (2008). In the present paper, the Differential Quadrature Method (DQM) [see, e.g., Wang et al. (2008), Wang and Ni (2009)] will be utilized to discretize the microtube. It has been demonstrated that the DQM is valid and efficient for analyzing the dynamical behavior of fluid-conveying tubes.

According to the basic idea of DQM, the domain x ($0 \leq x \leq L$) can be divided into N sampling points. The partial derivative of the displacement $w(x, t)$ with respect to x at a given discrete point x_j can be approximately expressed by a weighted linear sum of the values of $w(x_i, t)$ with $i = 1, 2, \ldots, N$. The DQM will be used here directly. The interested reader is referred to Bert and Malik (1996) for more details on this numerical method.

In the numerical calculations, it has been found that the results with $N > 15$ give sufficient accuracy. In the current work, therefore, the total number of sampling points will be chosen as $N = 17$. Using the DQM, Eqs. (19) and (20) can be transformed to an assembled form given by

$$\begin{bmatrix} [K_{bb}] & [K_{bd}] \\ [K_{db}] & [K_{dd}] \end{bmatrix} \begin{Bmatrix} \{w_b\} \\ \{w_d\} \end{Bmatrix} + \begin{bmatrix} [0] & [0] \\ [G_{db}] & [G_{dd}] \end{bmatrix} \begin{Bmatrix} \{\dot{w}_b\} \\ \{\dot{w}_d\} \end{Bmatrix} + \begin{bmatrix} [0] & [0] \\ [M_{db}] & [M_{dd}] \end{bmatrix} \begin{Bmatrix} \{\ddot{w}_b\} \\ \{\ddot{w}_d\} \end{Bmatrix} = 0, \tag{21}$$

in which the subscript b represents the elements associated with the boundary points (at the two ends of the tube), while d represents the remainder. Similarly as before, the dot denotes the derivative with respect to time.

For self-excited vibration of the microtube, the solution of Eq. (21) may be written as

$$\{w\} = \{\overline{w}\}\exp(\omega t), \tag{22}$$

where

$$\{\overline{w}\} = \{\{\overline{w}_b\}^{\mathrm{T}}, \{\overline{w}_d\}^{\mathrm{T}}\}^{\mathrm{T}}, \tag{23}$$

and $\{\overline{w}\}$ is defined as an undetermined function of vibration amplitude. $\mathrm{Im}(\omega)$ is the natural frequency of the microtube.

Substitution of Eq. (22) in Eq. (21) yields a homogeneous equation, which corresponds to the generalized eigenvalue problem

$$(\omega^2[M] + \omega[C] + [K])\{\overline{w}_d\} = \{0\}. \tag{24}$$

To obtain a non-trivial solution of Eq. (24), the determinant of the coefficient matrix must vanish, i.e.

$$\det(\omega^2[M] + \omega[C] + [K]) = 0. \tag{25}$$

Based on Eq. (25), the eigenvalues can be easily computed numerically. Thus, one obtains the eigenfrequencies of fluid-conveying microtubes for various parameter values.

4. Results

Before displaying some numerical results, we must address the fact that one has to determine the value of the material length scale parameter, l, for a specific type of material. Generally, different materials have different values of l. As reported by Lam et al. (2003), for the couple stress model used here, the characteristic length l of microscale structures is given by

$$l = \frac{b_h}{\sqrt{3(1-\mu)}}, \tag{26}$$

where b_h is a higher-order bending parameter with units of length. For epoxy beam-type structures, Lam et al. (2003) identified the higher-order bending parameter as $b_h = 24\,\mu\mathrm{m}$. Taking Poisson's ratio of epoxy as $\mu = 0.38$, one obtains a characteristic length of $l = 17.6\,\mu\mathrm{m}$. McFarland and Colton (2005) identified the higher-order bending parameter as $b_h = 32\,\mu\mathrm{m}$ (or $53.7\,\mu\mathrm{m}$) for polypropylene beams. For steel and aluminum thin plates, in an earlier study, Ellis and Smith (1968) obtained the higher-order bending parameter as $b_h \sim 10\,\mu\mathrm{m}$. The current work will not discuss further about how to obtain the values of b_h. For more details on this topic, the interested reader is referred to Lam et al. (2003) and Nikolov et al. (2007).

To illustrate the newly derived solutions of a pinned–pinned fluid-conveying microtube, numerical calculations have been performed. For convenience of illustration, the microtube considered here is taken to be made of epoxy (Lam et al., 2003), since the value of l has been given by Park and Gao (2006). The material properties of the microtube used in the numerical calculations are chosen to be $E = 1.44\,\mathrm{GPa}$, $\rho_f = \rho_p = 1000\,\mathrm{kg/m^3}$, $\mu = 0.38$, and $l = 17.6\,\mu\mathrm{m}$. For comparison purposes, it is assumed that $\alpha = d/D = 0.8$ and $L/D = 20$, where d and D are, respectively, the inner and outer diameters of the microtube.

The numerical results are shown in Figs. 2–5. In these figures, the first natural frequencies predicted by the current modified couple stress theory and by the classical theory are given, for various values of the outside diameter (D). Of course, similar diagrams can be constructed for higher-order natural frequencies (e.g., the natural frequencies in the second and the third modes), but they do not give further insight to the problem. The main reason is that the first (lowest) natural frequencies can represent the principal vibration characteristics of fluid-conveying microtubes.

From Figs. 2–5, it is noted that the flow velocity (V) is the variable parameter. It can be seen that, for $V = 0$, the natural frequencies predicted by the modified couple stress theory are about 1.93 times greater than those predicted by the classical beam theory when the outside diameter of the microtube is approximately equal to the material length scale

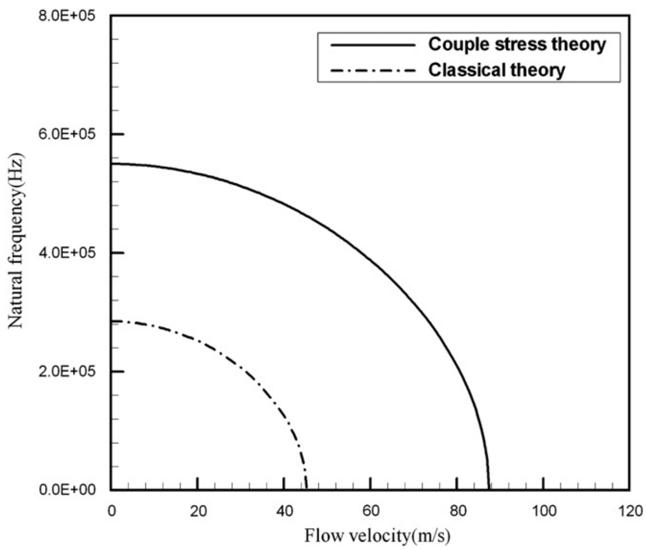

Fig. 2. Natural frequencies based on couple stress theory and classical theory, as functions of flow velocity, for the first mode of a pinned–pinned tube with $D = 20\,\mu m$.

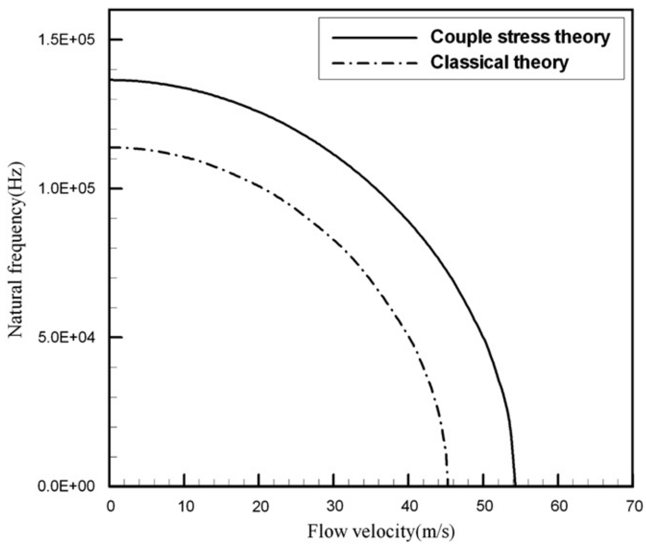

Fig. 3. Natural frequencies based on couple stress theory and classical theory, as functions of flow velocity, for the first mode of a pinned–pinned tube with $D = 50\,\mu m$.

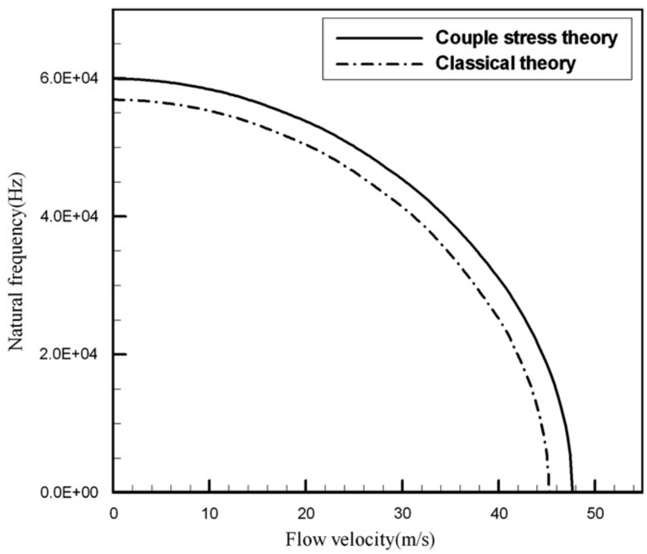

Fig. 4. Natural frequencies based on couple stress theory and classical theory, as functions of flow velocity, for the first mode of a pinned–pinned tube with $D = 100\,\mu m$.

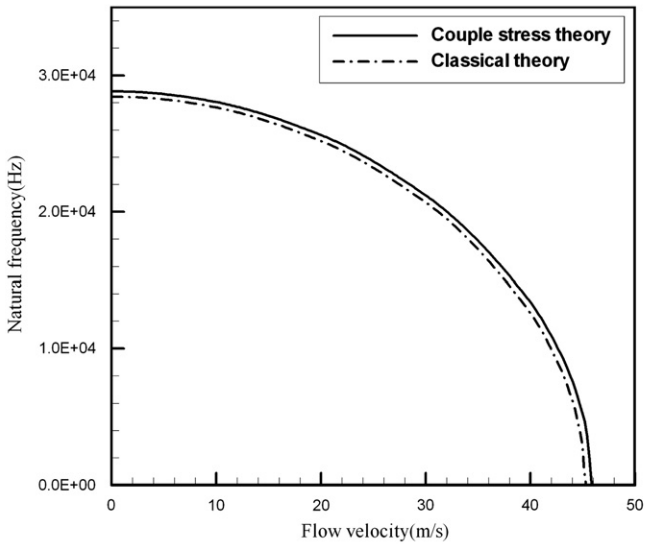

Fig. 5. Natural frequencies based on couple stress theory and classical theory, as functions of flow velocity, for the first mode of a pinned–pinned tube with $D = 200\,\mu m$.

parameter (i.e. $D = 20\,\mu m$). It is also shown that the difference between the two sets of values diminishes when the outside diameter of the tube becomes larger, hence indicating that the size effect is significant only when the outside diameter of the microtube is comparable to the material length scale parameter.

It is then of interest to see that the natural frequencies may become zero with increasing fluid velocity. This implies that the microtube would lose stability at a critical flow velocity (V_{cr}). The form of instability is divergence (buckling), since the system is conservative. From Figs. 2–5, it can be observed that the critical flow velocities predicted by the modified couple stress theory are generally higher than those predicted by the classical beam theory. Furthermore, the difference between the critical flow velocities predicted by the two models (present and classical) is found to be significant when the values of D/l are relatively small. It is also noted that the critical flow velocity predicted by the classical beam theory is $V_{cr} = 45.262\,m/s$. The available data in the literature indicate that the flow velocity inside nano- and microscale tubes might exceed hundreds of meters per second [see, e.g., Supple and Quirke (2003), Yoon et al. (2005)]. Thus, the flow velocity considered in the current work is within the practical range of flow velocity.

Regarding this point, it would seem that the internal material length scale makes the microtubes more stable, which favor the microtubes for micro-fluidic device applications. For small values of D/l (e.g., $D/l = 1.0$), the microstructure-dependent effect is strong and the proposed new tube model may be adequate for predicting the natural frequency and critical flow velocity of fluid-conveying microtubes. For large values of D/l (e.g., $D/l = 10$), the current tube model is not required and the classical tube model is acceptable, since the size effect may be negligible in that case.

Before leaving this section, it ought to be mentioned that Eq. (19) may be rewritten in its dimensionless form

$$(1+\gamma)\frac{\partial^4 W}{\partial \xi^4} + v^2\frac{\partial^2 W}{\partial \xi^2} + 2\beta^{1/2}v\frac{\partial^2 W}{\partial \xi \partial \tau} + \frac{\partial^2 W}{\partial \tau^2} = 0, \tag{27}$$

where

$$\xi = x/L, \quad W = w/L, \quad \tau = \left[\frac{EI}{m+M}\right]^{1/2}\frac{t}{L^2}, \quad \beta = \frac{M}{M+m}, \quad v = \left[\frac{M}{EI}\right]^{1/2} LV, \quad \gamma = \frac{8}{(1+\mu)(1+\alpha^2)\chi^2}, \quad \chi = \frac{D}{l}, \tag{28}$$

for circular microtubes.

Therefore, the current theoretical model can be used to analyze the dynamics of microscale tubes containing internal fluid, regardless of the tube material or length scale. For a specific tube material, the dynamics can be predicted by the present tube model, provided that the value of l has been obtained.

5. Conclusions

In this paper, a new equation of motion has been derived for the vibration and stability of fluid-conveying microtubes using Hamilton's principle. The tube material is assumed to obey the modified couple stress theory as developed by Yang et al. (2002). The microtube model contains an internal material length scale parameter in addition to four classical constants of the tube system.

Based on the derived equation of motion, the vibration characteristics of the system were studied, and the existence of a divergent instability of the microtube was demonstrated. In addition, the effects of the internal material length scale parameter on the natural frequencies and on the critical flow velocities were explored. It is found that both natural frequencies and the critical flow velocities predicted by the modified couple stress theory are larger than those predicted by the classical beam theory. The difference between the results obtained by these two theories is significant when the characteristic size (i.e. the diameter of the microtube) is comparable to the material length scale parameter, but decreases with increasing characteristic size.

Finally, it should be mentioned that, like all other analytical models, the newly developed tube model has limitations, which are contingent on the applicability of the modified couple stress theory. Specifically, the tube must undergo small deformations so that the linear geometrical relations given in Eqs. (3) and (5) are applicable. Additionally, the tube material must be isotropic, homogeneous, and linearly elastic, in order for the linear constitutive relations listed in Eqs. (2) and (4) to remain valid. In particular, the material length scale parameter, l, has to be identified before using the current model. As previously stated, different materials have different values of l. For microtubes made of a specific material with sufficiently small l (e.g., l is at the nanoscale), the value of D/l may be large enough that the microstructure-dependent effect is not observable.

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